Co-funded by the
Erasmus+ Programme
of the European Union

ViduKids Case Studies

Norway

Ducks divide money – problem-solving in Kindergarten

Oliver Thiel and Anne Hj. Nakken[1]

In this case study (Thiel & Nakken, 2022), we explored how animation video production supports young children's mathematical problem-solving.

Introduction

Adults often think mathematics is mainly about numbers and solving tasks with given methods (Thiel, 2010, p. 112f.). That is not the case for both younger children and professional mathematicians. For them, the focus is on exploring patterns and structures, discovering connections and solving problems (Krummeck & Richter-Gebert, 2005, p. 1; Nakken & Thiel, 2019, p. 24). Non-mathematicians often find mathematics

[1] This case study has previously been published in German: Thiel, O. and Nakken, A. Hj. (2022). Kindergartenkinder produzieren mathematische Videos. Erste Ergebnisse aus dem Projekt ViduKids. In M. Gutzmann and U. Carle (Eds.), *Anfangsunterricht für alle Kinder - Willkommen in der Schule!* Grundschulverband : Frankfurt am Main, pp. 226-245.

difficult because it is abstract (Devlin, 2012, p. vii). Piaget (1975, p. 361) assumed that children are not capable of abstract logical reasoning before they reach the formal operational stage, around the age of 7 years. Recent research (Clements & Sarama, 2009; Fuson, 1992; Sarama & Clements, 2009), however, shows that abstract thinking begins much earlier. Playing is crucial in transitioning from concrete to abstract thinking (Poland & van Oers, 2007). Otsuka and Jay (2017, p. 998) found three properties of game situations that promote the transition from concrete to abstract thinking:

1. The children share their thinking with other children and adults.
2. The children chose to pause to reflect on their experiences.
3. The children demonstrate satisfaction with the result of their self-directed play.

One of the ViduKids project aims is to create play situations that meet these criteria with the help of video technology.

Context, approach & Implementation

Problem-solving

Problem-solving is not only an important goal of mathematics lessons but also one of the most important methods of learning mathematics (NCTM, 2000, p. 52). Even small children learn in Kindergarten that mathematics helps solve problems of all kinds (Björklund, 2012, p. 158). Whether a situation is perceived as a problem or a task depends on the learners' previous experiences. To solve a task, the learner must just apply a known method correctly. On the other hand, problem-solving is what you do when you don't know what to do (Wheatley & Wheatley, 1984, p. 22). For most fourth-graders, 20 divided by 4 is an easy task. For children in preschool or Kindergarten, it can be a complex problem to solve.

Mathematical problems for kindergarten children should have the following six properties (Ramírez-Uclés et al., 2018, p. 577):

(1) **Reasoning**. The problem must explore and develop mathematical ideas through reason, the use of strategies, as well as several trial-and-error cycles rather than through algorithms.

(2) **Contexts**. The problem must refer to situations familiar to the child. Situations need not necessarily be real from an adult's standpoint: stories, TV shows, and cartoon series are also acceptable.

(3) **Challenge**. The problem must motivate the child to seek a solution. That may be furthered with different representations (verbal, physical and graphic), requiring the child to handle, transform or modify materials.
(4) **Multiple solutions**. The problem must afford different levels of solutions, which must consist of more than short answers.
(5) **Expandability**. The mathematical structure must be applicable to a number of situations to enable children to generalise.
(6) **Comprehensibility**. The problem must be understandable for all children, who must be convinced that they can solve it and know when they have found the solution.

Sharing as a problem in Kindergarten

There are two different approaches to division: measuring and distributing.

- When **measuring**, the dividend is broken up into equally sized parts. The divisor indicates how large each piece (the unit) is. The result is the number of parts. Measuring can be realised by repeatedly removing the divisor from the dividend – until nothing or only a remainder is left.
- When **distributing**, the dividend is divided evenly among a specified number of subsets. The divisor indicates the number of subsets. The result is the number of elements in each subset. The distribution can be realised by a repeated one-to-one assignment – until nothing or only a remainder is left. If the dividend is sufficiently large, bundles of two or three can be distributed.

In the case of everyday problems, the situational context determines which approach is appropriate. The well-researched mathematical problem for kindergarten children is a distribution situation that can be varied easily. Two or three dolls shall share six or twelve biscuits (Davis & Pepper, 1992, p. 400), three or four pirates shall share twelve gold coins (Ramírez-Uclés et al., 2018, p. 578), or four or five plush rats shall share twenty coins (Justnes, 2018, p. 9).

Many four- and five-year-olds, and even some three-year-olds, successfully distribute when they share countable objects fairly (Davis & Pepper, 1992, p. 398). Perhaps they use this strategy without realising that it is not only practical but also ensures fair distribution. Even many second graders count at the end to ensure they accomplished the task (Davis & Pitkethly, 1990, p. 153). Most kindergarten children can mechanically distribute twelve biscuits to two dolls without further thought about the process. When a third doll is added, so the biscuits have to be redistributed, a real problem arises for most children (Davis & Pepper, 1992, p. 401). In the study by Davis and Pepper (1992, p. 404), only 61 % of the children could solve this problem. The 45 children who solved it used 26 different strategies, most of which did not require counting skills. The most common strategy was also the most efficient. It was used by 20 % of the children: take two biscuits from the first doll and give them to the third. Then take two biscuits from the second doll and provide them to the third (Davis & Pepper, 1992, p. 410).

Implementation

As a mathematical problem, we chose 20 divided by 4 and 20 divided by 5 as described by Justnes (2018, p. 8), but with rubber ducks instead of plush rats, since Anne, the teacher involved, has an extensive collection of different rubber ducks.

The activity was conducted on 4th May 2021, first with a small group of three children aged four to six years and then again with two children aged four and five. The activity had four phases.

1. Anne playfully presented the mathematical problem.
2. Supported by Anne, the children produced a Stop motion video that shows the solution to the problem.
3. The children watched the final video and added sound to it.
4. The whole kindergarten group watched the video together and reflected on it.

Phases 1 to 3 lasted 60 minutes in the first group and 30 minutes in the second. Phase 4 was conducted in the morning circle the following week. As part of the case study, we interviewed the children before phase 1 about their previous experiences with digital technology and mathematics. After phase 3, we asked them about their opinion of the activity. All children participated in the activity voluntarily and with their parent's permission. The observations were recorded on video.

Challenges

We observed more pedagogical than technical challenges. All children said that they had used a smartphone before to view videos or pictures, but also to take photos or videos. Thus, it was not challenging for them to take the images for the stop motion video. It was challenging, however, to take a picture before moving the objects. Often the children were so engaged in the play that they forgot to take pictures. Playing was more important to the children than making a video or solving a problem. It was challenging for the teacher to guide them back to the problem and the filming task without disturbing the children's play. Play is essential to early childhood education and the children's main activity (Øksnes, 2017).

Results

When asked what they know about sharing, they answer as follows:

> Alina (6): 'You must ask nicely and wait for the answer. You have to say thank you afterwards'.
>
> Robert (5): 'You can divide into quarters'.
>
> Nora (4): 'You have to share so that everyone gets something'.
>
> Matheo (4): 'You have to share chairs. Put things together'.
>
> Emilian (5): 'First everyone gets two, and if there are more, you get two more. If it's not enough that everyone gets two, you have to buy more'.

This shows that the children had very different experiences sharing in everyday life.

1. The children solve the problem

Anne introduced the problem to the children and explained that they could make a video about it. She sat on the floor with three children and placed a queen duck between them. Then she started counting coins. The children helped her count to 20. 'The queen has 20 coins', Anne said and took three more ducks out of her pocket – one for each child. She played that the ducks asked the queen to share the money with them. Then she asked the children to help.

Nora started distributing the coins to the ducks one-by-one, but Robert said, 'I want to do it myself'. So all children collected coins for their ducks. Anne reminded them not to forget the queen. Finally, Robert stated, 'I have five'. The girls counted, too. They and the queen also had five coins each (see Fig. 1). Anne asked how they solved the problem. Alina replied, 'We counted', but Robert said, 'I got five because I am 5'.

Fig. 1: The children distributed 20 coins to four ducks

The observation supports the theory that preschool children can distribute but are unaware that one-to-one correspondence is an effective strategy (Davis & Pepper, 1992, p. 398). They believe that counting is necessary to ensure fair sharing (Davis & Pitkethly, 1990, p. 153).

Anne expanded the problem by introducing another duck who wanted coins, too. The children did not like to share, but after some hesitation, thinking, and discussion, the children moved one coin from each duck to the new duck. They counted and agreed that each duck now had four coins.

Contrary to the results of Davis and Pepper (1992, p. 404), redistribution was not that difficult for the children. Even the 4-years-old could find the solution. However, the transition from 20/4 to 20/5 is more straightforward than from 12/2 to 12/3 because everyone has to discard only one coin, not two.

Robert suggested that even more ducks might join in. Anne made a clown duck appear and asked hypothetically, 'What happens if everyone gives him a coin?' Robert replied, 'Then we have three'. Anne asked, 'Will each duck have three then?' The children did not answer but played with the ducks. Anne asked them to try it. Everyone except Nora and Robert gave a coin to the clown. Now, four ducks had three coins each, but Nora and Robert's ducks still had four each. Anne asked if everyone could get the same number. Alina explained, 'No. They have three each over there, and four over here', and Robert added, 'If we give a coin, you have four, and we only have three'. 'That's strange', Anne said and ended the activity because the children only talked about what they wanted to buy with the money.

The children were able to transfer the strategy for solving the first redistribution problem to the new situation and realised it was not successful here. They seemed to accept that the problem was unsolvable and lost interest. In addition, it seems less important that the sharing is fair if you are the one who got more than the others did.

Anne presented the problem similarly in the second group, which consisted of two boys, but the children reacted differently. Matheo (4) did not know what to do. Emilian (5) first distributed two, then two more to each of the four ducks. When Matheo saw this, he also started distributing (see Fig. 2). Suddenly each duck had five coins. Anne asked how many coins each duck had. Matheo started counting, but he counted some coins several times. Anne interrupted and showed him to touch and move the coins while counting them. In that way, the children found out that each duck had five coins. Anne decided to skip the redistribution problem.

Fig. 2: Matheo and Emilian distribute coins

2. The children are filming

A table with two white A2 sheets of paper served as the stage for the video. For the filming, Anne used her iPhone with a small tripod. She explained to the children what had to be in the first picture and asked Robert to arrange it: the queen and 20 coins. The children had to count. Robert first tried to count the coins lying in a heap. When that failed, he decided to push each coin aside as he counted. Anne then showed them how to take a picture and asked what should be on the following image. The children played with their ducks, begging the queen for money. When Alina started distributing the coins, Anne stopped her and explained that things had to be slower. Robert was allowed to put his duck on the table and take a picture, then Alina and finally Nora. Then Anne said, ‘Now we have to share the money’. The children started playing and distributing the coins. Anne had to stop them again, ‘Not so fast! We have to take pictures’. Alina asked about a feature of the stop motion app, and Anne explained it. Then she asked Robert to move a coin to the blue duck and take a picture. Robert did it (see Fig. 3), and Anne asked how to proceed. The children decided that the blue duck should get five coins first, then the red one and finally the green one. The last five coins stayed with the queen.

Fig. 3: The children take pictures

Interestingly, the children did not use the same strategy as previous but remembered the solution that each duck gets five coins.

Anne asked if there should be another duck. The children answered yes, and Nora got another red duck. Anne asked Nora, ‘Do you remember how we did it?’ Nora replied, ‘We returned all the money to the Queen’. Anne was surprised, ‘And then redistributed it? Is that how we did it?' Robert replied, 'No, we gave him some money', pointing at the new duck. Anne asked, 'Do you remember, Nora, how many we gave him?' Nora replied by moving a coin from her duck to the new one. Anne asked her to take a picture. Then Nora explained the whole process while distributing the coins and Anne taking photographs. When everything was finished, Anne decided to skip the additional problem with the sixth duck.

Although Nora remembered the original strategy when asked, she first suggested a new one. Reducing a problem to an analogous case for which a solution already exists is a popular problem-solving strategy in mathematics. In the study by Davis and Pepper (1992, p. 401), this strategy was relatively rare. Only 5% of all children used it. Among the good counters, however, it was the most frequent strategy (Davis & Pepper, 1992, p. 408).

The video made by the first group has 36 frames and is 18 seconds long when played at two frames per second. A stop motion video should have at least ten frames per second to obtain smooth movements. Since the preschool children were more engaged in playing the story than taking pictures, they did not accomplish that many. Nevertheless, 36 frames were enough to show the solution to the problem, and the children were delighted with the result.

In the second group, the filming went differently than in the first. Emilian began distributing the coins to the ducks in pairs as before. The boys took turns. When each duck had four coins, four coins remained undistributed. Anne asked what they could do with these. Instead of distributing the coins one-by-one to the four ducks, Emilian suggested getting another duck. He changed the problem creatively instead of solving it. Apparently, Emilian could not remember the original previous solution that they accomplished due to Matheo's intervention. We do not know why Emilian believes you can only distribute in pairs. Finally, the remaining four coins were given in pairs to the new duck (see Fig. 4).

Fig. 4: Matheo gives two coins to the fifth duck

The second group's video is eleven seconds long. It has 22 frames, but some are repeated shots of the same situation. Only 14 images are different and correspond approximately to the 11 steps of the problem-solving.

3. The children add sound to the video

The stop motion technique produces a silent movie. Sound can be added in post-production. The children's videos are too short to include dialogue between the ducks. Therefore, Anne decided that the children should explain or describe the solution to the problem. By doing that, they had to reflect again on how they solved the problem. First, however, Anne had to show them how to make a sound recording. In the first group, she demonstrated it to the children as an example. As a result, the children mostly repeated what she had said. Alina said, 'Once upon a time, there was a queen. Then a blue, a green, and a pink duck joined in. And then they divided and divided. And then another pink duck joined them. And they divided a lot'. Robert (5) said, 'Once upon a time, there was a queen. She had 20 coins. She shared it with the blue and the green, and the pink. And she said, "No, no, no, don't use the money!" And then she took the money back. And so they just divided and divided, and then' – the recording stopped. Both children told the story instead of explaining how they solved the problem (see Fig. 5). We suspect the children did not experience the activity as a mathematics problem. Alina mentioned the appearance of the fifth duck, which created the redistribution problem. Robert did not get that far in his story before the recording stopped. As in phase 1, he was mainly concerned with not wanting to share.

Fig. 5: Robert adds his voice to the video

In the second group, Anne avoided giving the children an example explanation. Matheo only briefly summarised the most important fact, 'Four each'. Emilian said, 'There are many each. And there are four. And there are ducks, too'. He neither told the story nor explained the problem-solving process. He described the solution. Both children were exclusively fixated on the solution. Preschool children are not expected to have an operational understanding of division. One-to-one correspondence, distributing and counting are operations preschool children should gain experience with. During the ViduKids activity, it became clear that Matheo lacked an adequate operational understanding of all three operations. This gave Anne essential clues as to how she could support him in his counting development.

4. The kindergarten group watches the videos

The following week Anne showed the videos to the other children in the Kindergarten. Anne chose not to make any comments before the videos were shown. The videos alone should be the starting point for the subsequent discussion. When the first video started, one of the four-year-olds called out, 'That's Emilian!' He recognised it from the voice. The other children listened carefully and quickly agreed that they heard Emilian.

Emilian expressed embarrassment and pride at the same time with his body language. Blushing slightly, he curled up in the chair, nodding in agreement with a smile. Anne played the video several times and then paused to listen to the children's mathematics comments. Sofie (4) repeated Emilian's statement from the video, 'Four for each', and showed four fingers with her hand. Several children then held up four fingers, too. 'They trade money to make it fair', Tuva said. The other children agreed. Anne replayed the video and stopped at the frame where all the coins were distributed. The children immediately started touching the screen and counting the coins. Anne heard that Sofie counted to four, Tuva to five, Viktoria to eight. Anne encouraged Sofie to show how she counted. She counted, '1, 2, 3, 4' by touching the coins in front of the green duck one by one. Anne challenged Tuva to demonstrate her counting but was interrupted by Olivia (4). Olivia wanted everyone to count the number of coins in front of all the ducks. Together Anne and the children counted five times (once for each duck), '1, 2, 3, 4'. Then Tuva could finally show how she had counted. She counted, '1, 2, 3, 4, 5' while touching the ducks on the screen. Anne and the children agreed that there were five ducks, each receiving four coins. To direct the children's attention to the distribution strategies, Anne asked, 'How did the ducks share?' Tuva answered by repeating her previous statement, 'They trade money to make it fair'. These observations show that preschool children spontaneously focus on numerosity (SFON), but spontaneous focusing on quantitative relations (SFOR) and multiplicative structures develop later (Hannula-Sormunen et al., 2019, p. 29 and 34).

Anne started the next video. The children found it amusing that Robert in the video gave his voice to the queen, shouting, 'No, no, no'. Some of the children laughed and imitated this. Anne asked again how the ducks distributed the money. The children quickly realised that the ducks received four coins each as well. They were about to move on to the next video, but Anne replayed the video and stopped at the frame where the coins were distributed, asking the children if they wanted to count the coins. Tuva quickly replied, 'No need for that. I can see it is the same'. Perhaps Tuva was subitising the numbers (Hannula-Sormunen et al., 2019, p. 26).

Finally, Anne showed the video dubbed by Matheo. Matheo sat perfectly still during the performance and smiled. The children already knew that each duck finally receives four coins. Therefore, they considered this video as less exciting. Anne asked what all three videos were about. Tuva quickly replied, 'about ducks and money. There was a hundred money, and they traded in such that each got four.' Olivia added, 'That was fair.' Tuva used 'hundred' here as a synonym for 'many' and not as an exact number (Nakken & Thiel, 2019, p. 175). We do not know why she said trade instead of share.

Later that same day, there was an argument between some girls who were playing with Barbie dolls. They could not agree on the distribution of the dresses to the dolls, as two girls wanted the same dress. Then Olivia said sternly, 'Remember, we have to share. So did the ducks, and then it was fair.' Although Olivia has not yet mastered all the aspects of the mathematical operation division, she has learnt important lessons about the social and ethical aspects of sharing through her involvement with the videos. This corresponds to the holistic learning that is inherent in kindergartens in countries that have a social pedagogy tradition (Bertram & Pascal, 2016, p. 110).

Evaluation/Reflection/Discussion

The observations revealed both strengths and weaknesses of the ViduKids approach. The ducks invited the children to play more than to solve the problem. However, this is not a disadvantage but an essential element of early childhood education. In kindergarten, play is the children's main activity (Fröbel, 1887, p. 57; Øksnes, 2017). In the final interview after phase 3, all children said that the activity was fun and that they would like to do it again some other time. Two children answered, 'because we could play with money' and 'because we could play with the ducks'. The three other children replied, 'because we could click on the smartphone', 'because we were allowed to use the smartphone', and 'it was exciting to see what the video would be like'. Play and technology were most important for the children. When asked what they had learnt, none of the children could answer. Does that mean they have not learnt anything? We do not think so. Kindergarten teachers do not talk about learning. Therefore, it is difficult for the children to reflect on their learning processes. Furthermore, learning mathematics is a cumulative process. The children gather experiences that slowly lead to a deeper understanding of mathematical relationships.

The activity exhibits all six properties that math problems for kindergarten children should have (Ramírez-Uclés et al., 2018, p. 577):

(1) **Reasoning**. The children were able to solve parts of the task mechanically. However, sometimes they had to stop, think, and try different approaches. They argued very little verbally but expressed their thoughts through their actions.
(2) **Contexts**. Sharing was familiar to the children.
(3) **Challenge**. The problem was motivating, such that the children solved it with enthusiasm. The coins provided allowed them to find the solution enactively.
(4) **Multiple solutions**. It became clear that the problem actually allowed for various solutions. The two groups of children solved the problem differently. Even within each group, the children suggested different solutions during the filming than they had found in the first phase.

(5) **Expandability**. In the distribution situation the children used familiar strategies. Redistributing gave them new experiences that will come in handy when faced with similar problems. They expanded not only their mathematical skills but also their social skills related to fairness and sharing.
(6) **Comprehensibility**. Except for Matheo, all children understood the verbal description of the problem. Matheo understood the issue when he saw Emilian's approach. During the filming, it became clear that Matheo had contributed to solving the problem in phase 1. All the children except Matheo could count the coins to prove they had solved the problem. Matheo succeeded with the help of the teacher. Thus, counting is in Matheo's zone of proximal development (Vygotsky, 1978).

To a certain extent, the activity also fulfilled the three properties of game situations that promote the transition from concrete to abstract thinking (Otsuka & Jay, 2017, p. 998):

1. The children shared their thoughts with other children and the teacher. We observed several times in phases 1 and 2 that the children shared their ideas through their actions. Later in phase 4, the children shared their thoughts both verbally and through gestures, such as tapping on the screen. Anne's open questions encouraged the children to share their thoughts (Nakken & Justnes, 2018, p. 45).
2. The children took time to reflect on their experiences. During our observations, there were no pauses in which no one spoke. The children, however, showed clear signs of thinking while the others were talking. An example is Nora at the beginning of the redistribution problem. After Alina gave a coin to the new duck, Nora looked at the money and the ducks for 18 seconds without saying anything. Then Anne asked, 'Nora, what should we do?' It took another 5 seconds before she finally moved one of her coins to the new duck. Afterwards, Robert and Nora showed for 20 seconds with their body language that they were thinking (see Fig. 6).

Fig. 6. The children reflect on the problem

3. The children expressed satisfaction with the results when they solved a problem (see Robert's expression in Fig. 1) and with the final videos. Bennett et al. (1997, p. 13) observed that children direct their learning effectively when they can control their activities autonomously. In our activity, the children's autonomy was limited because the problem was given. However, the children had much freedom in solving the problem – even finding unconventional solutions. This has contributed to the children's satisfaction. The children had experiences that were appropriate for them. This became particularly clear when adding sound to the videos. The children who found the mathematics easy to solve focused on telling the story, while the children for whom it was a genuine mathematical problem described the mathematical solution.

Conclusion

In this case study, we analysed and discussed observations to provide preliminary answers to our research question. How does video production in Kindergarten contribute to preschool children having varied mathematical experiences, discovering and understanding mathematical relationships and developing a positive attitude towards mathematics?

The first thing we can state here is that four- to six-year-olds can produce Stop-Motion videos with the support of an adult. Even though the videos are not very long, they convey the main parts of the story that the children want to tell. Furthermore, the Stop-Motion technique helps the children understand that a motion picture is made up of many individual frames that only give the impression of movement when played back in quick succession.

Second, our observations clearly show that the method allows children to engage with the same problem four times naturally and playfully when (1) solving the problem, (2) filming, (3) adding sound, and (4) watching the video. In each phase, the children can have new experiences, recognise other possible solutions, and gain deeper insights. However, the extent to which this happens varies from child to child.

All the children enjoyed the activity. This helps develop a positive attitude towards the creative use of digital media. If this also creates a positive attitude toward mathematics depends on how aware the children are that they are doing mathematics. Many preschool teachers avoid talking about mathematics in Kindergarten. Jenßen (2021, p. 90) found a correlation between fear of mathematics and career choice to become a preschool teacher. In this respect, we hope that ViduKids will help not only the children but also the kindergarten staff to develop a positive attitude towards mathematics.

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